



A Finite Groups Package

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Introduction

As part of our MITACS research project at SFU we are developing a library of and commands for creating finite groups in Maple. This allows group theorists to use Maple for computations on finite groups. This package contains:

- 1) A finite presentation of all non isomorphic small groups of order up to 200.
- 2) A permutation representation of classical matrix groups of moderate size.
- 3) Finite simple groups including alternating groups, classical and exceptional groups of Lie type, and sporadic groups.
- 4) Functions for creating groups of particular classes such as: cyclic, symmetric, alternating, dihedral, dicyclic and metacyclic.

The storage size of the library is 13 MB.

Small Groups

Contains all groups of small orders up to 200 (see [1] and [3]). The groups are sorted by their orders and they are listed up to isomorphism; that is, for each of the available orders a complete and irredundant list of isomorphism type representatives of groups is given. These groups are available as finitely presented groups.

Classical Matrix Groups

Contains a permutation representation of the general linear, special linear, orthogonal, special orthogonal, unitary and symplectic groups of moderate size, including their projective groups.

Simple Groups

Every finite simple group to be one of the following types:

- A cyclic group with prime order.
- An alternating group of degree at least 5.
- A classical group (projective special linear, symplectic, orthogonal or unitary group over a finite field).
- An exceptional or twisted group of Lie type (including the Tits group).
- One of 26 left-over groups known as the sporadic groups.

These groups are listed in the Atlas of Finite Groups [2] (see also [4]).

Classical Simple Groups

Contains a permutation representation of moderate size classical simple groups and some twisted groups of Lie type which are,

- Chevalley groups $G_2(q)$, $F_4(q)$, $E_6(q)$, $E_7(q)$, $E_8(q)$,
- Ree and Suzuki groups $R(q)$, $Sz(q)$,
- Steinberg groups ${}^3D_4(q)$, ${}^2E_6(q)$ and
- Tits group $T = {}^2F_4(2)'$.

Sporadic Groups

Provides a permutation representation of some sporadic simple groups. These are

M_{11} , M_{12} , M_{22} , M_{23} , M_{24}	Mathieu groups
J_1 , J_2 , J_3	Janko groups
HS	Higman-Sims group
McL	McLaughlin group
HE	Held group
Co_2 , Co_3	Conway groups
Suz	Suzuki group
Fi_{22}	Fischer group
Ru	Rudvalis group

Some Standard Finite Groups

This package has functions for creating some standard finite groups such as alternating, symmetric, cyclic, dicyclic, metacyclic and dihedral groups.

Examples

Small Groups:

```
> MrSmallGroups(192);
1543
> SmallGrelgroup(4, 2);
grelgroup([a1, a2], [[a1, a1], [1/a2, 1/a1, a2, a1], [a2, a2]])
> AllSmallGrelgroups(9);
[grelgroup([a1, a2], [[1/a2, 1/a1, a2, a1], [a2, a2, a2], [a1, a1, a1, 1/a2]]),
grelgroup([a1, a2], [[a1, a1, a1], [1/a2, 1/a1,
```

Permutation Representation of Classic Matrix Groups:

```
> GeneralLinearGroup(3, 3);
permgrou on 26 symbols with 2 generators
> ProjGeneralUnitaryGroup(5, 2);
permgrou on 341 symbols with 2 generators
> SymplecticGroup(4, 3);
permgrou on 80 symbols with 2 generators
```

Simple Groups:

```
> AlternatingGroup(7);
permgrou(7, ([[1, 2, 3, 4, 5, 6, 7]], [[5, 6, 7]]))
> ProjSpecialLinearGroup(2, 11);
permgrou(12, ([[1, 2, 8], [3, 7, 9], [4, 10, 5], [6, 12, 11]], [[3, 11, 9, 7, 5], [4, 12, 10, 8, 6]]))
> OrthogonalGroup(`O8+(2)`);
permgrou on 120 symbols with 2 generators
> ExceptionalGroup(`Sz(8)`);
permgrou on 65 symbols with 2 generators
> SporadicGroup(`Co3`);
permgrou on 276 symbols with 2 generators
```

Other Finite Groups:

```
> CyclicPermgrou(19);
permgrou on 19 symbols with 1 generators
> DihedralGrelgroup(4);
grelgroup([a, b], [[a, b, a, b], [b, b], [a, a, a]])
```

Commands

The following is the list of available commands in this package.

AllSmallGrelgroups
AllSmallPermgrou
AlternatingGroup
CyclicGrelgroup
CyclicPermgrou
DicyclicGrelgroup
DicyclicPermgrou
DihedralGrelgroup
DihedralPermgrou
ExceptionalGroup
GeneralLinearGroup
GeneralOrthogonalGroup
GeneralUnitaryGroup
MathieuGroup
MetacyclicGrelgroup
MetacyclicPermgrou
NrSmallGroups
OrthogonalGroup
ProjGeneralLinearGroup
ProjGeneralUnitaryGroup
ProjSpecialLinearGroup
ProjSpecialUnitaryGroup
ProjSymplecticGroup
SpecialLinearGroup
SpecialOrthogonalGroup
SpecialUnitaryGroup
SporadicGroup
SymmetricGroup
SymplecticGroup

References

- [1] H.U. Besche, B. Eick and E. O'Brien, The Small Groups library, 2002. (<http://www-public.tu-bs.de:8080/~beick/soft/small/small.html>)
- [2] J.H. Conway, R.T. Curtis, S.P. Norton, R.A. Parker and R.A. Wilson, *Atlas of Finite Groups. Maximal Subgroups and Ordinary Characters for Simple Groups*, Clarendon Press, Oxford, 1985.
- [3] The GAP Group, GAP -- Groups, Algorithms, and Programming, Version 4.4; 2006. (<http://www.gap-system.org>)
- [4] R.A. Wilson et al., *ATLAS of Finite Group Representations - Version 3*; 2006. (<http://brauer.maths.qmul.ac.uk/Atlas/v3/>)

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